$K-\bar{K}$ mixing

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Both K^0 and $\bar{K}^0$ should mix into each other. A simple reason is that both decay to two pions,

We can write the wavefunction

$$|\psi(t)\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \equiv a(t)|K^0\rangle + b(t)|\bar{K}^0\rangle$$

In the Schrödinger picture, we evolve it by

$$i\frac{d}{dt}|\psi(t)\rangle = \left(M - \frac{i}{2}\Gamma\right)|\psi(t)\rangle$$

where M and Γ are the parts of the self-energy with $K^0, \bar{K}^0$ as internal states (mass) and others (decay)

Unitarity requires M and Γ to be hermitian, $M^\dagger = M$, $\Gamma^\dagger = \Gamma$.

Therefore,

$$M_{12} = \frac{\Sigma_{12} + \Sigma_{21}^*}{2} \quad , \quad \frac{\Gamma_{12}}{2} = i\,\frac{\Sigma_{12} - \Sigma_{12}^*}{2}$$

while M_{11} and M_{22} depend on the strong interactions (they are the mass terms in the chiral lagrangian), the off-diagonal terms M_{12} and the decay matrix Γ depend on the weak interactions and are much smaller.

However, CPT imposes $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, so the eigenvalues of Σ are degenerate for $\Sigma_{12} = 0$. So even if Σ_{12} is small, the degeneracy is lifted and leads to meson-antimeson mixing.

The K^0 & $\bar{K}^0$ states are related by

$$CP|K^0\rangle = \xi_K |\bar{K}^0\rangle \quad , \quad |\xi_K|^2 = 1.$$

we take the convention $\xi_K = -1$. We have

$$\begin{aligned}\langle K^0 | H_{eff} | \bar{K}^0\rangle &= \langle K^0|(CP)^{-1} CP\, H_{eff}\, (CP)^{-1} CP\, |\bar{K}^0\rangle \\ &= \langle \bar{K}^0 | H_{eff} | K^0\rangle\end{aligned}$$

if CP is preserved. So CP conservation

leads to $\Sigma_{12} = \Sigma_{21}$. Together with hermiticity, it would imply that both M_{12} and Γ_{12} are real.

If CP is not preserved (as in our world) we have the eigenstates

$$|K_L\rangle = \frac{1}{\sqrt{|p|^2+|q|^2}}\left(p|K^0\rangle + q|\bar{K}^0\rangle\right)$$

$$|K_S\rangle = \frac{1}{\sqrt{|p|^2+|q|^2}}\left(p|K^0\rangle - q|\bar{K}^0\rangle\right)$$

where

$$\frac{p}{q} = \sqrt{\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}} \quad , \quad M_{12} - \frac{i}{2}\Gamma_{12} = \langle K^0|H|\bar{K}^0\rangle$$

the mass splitting, $A \pm pq$, is given by

$$\begin{aligned} 2pq &= (m_L - m_S) - \frac{i}{2}(\Gamma_L - \Gamma_S) \\ &= 2\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)^{1/2}\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)^{1/2} \\ &\simeq 2\,\mathrm{Re}\,M_{12} - i\,\mathrm{Re}\,\Gamma_{12} \end{aligned}$$

where we assumed that CP violation is small, $\mathrm{Im}\,M_{12}/\mathrm{Re}\,M_{12} \ll 1$.

The "L" and "S" subscripts stand for

long and short, and refer to the lifetimes of the states. The ratio is actually large,

$$\tau_L / \tau_S \simeq 571$$

If CP was conserved, $p = q$ and the states would become CP eigenstates,

$$|K_S\rangle \to |K^0_+\rangle = \frac{1}{\sqrt{2}} \left(|K^0\rangle - |\bar{K}^0\rangle \right)$$
$$|K_L\rangle \to |K^0_-\rangle = \frac{1}{\sqrt{2}} \left(|K^0\rangle + |\bar{K}^0\rangle \right)$$

$$CP |K^0_\pm\rangle = \pm |K^0_\pm\rangle$$

In this limit, K_S would decay only to CP-even states like $\pi\pi$, and K_L would decay to CP-odd states like $\pi\pi\pi$.
Since the phase space for K to decay to 3π is small, K_- will have a much larger lifetime.

. strong interactions produce either K^0 or $\bar{K}^0$. Then the state evolves via

$$|K^0(t)\rangle = g_+(t) |K^0\rangle + \frac{q}{p} g_-(t) |\bar{K}^0\rangle$$
$$|\bar{K}^0(t)\rangle = \frac{p}{q} g_-(t) |K^0\rangle + g_+(t) |\bar{K}^0\rangle$$

with
$$g_\pm(t) = \frac{1}{2} e^{-\Gamma_L \frac{t}{2}} e^{-i m_L t} \left[1 \pm e^{-\Delta\Gamma \frac{t}{2}} e^{i \Delta m t} \right]$$

$$; \ \Delta\Gamma = \Gamma_S - \Gamma_L > 0$$
$$\Delta m = m_L - m_S > 0$$

By observing the oscillations of the decay products of both K^0 & $\bar{K}^0$, one determines

$$\Delta m_{exp} = (3.484 \pm 0.006) \cdot 10^{-12} \text{ MeV}$$

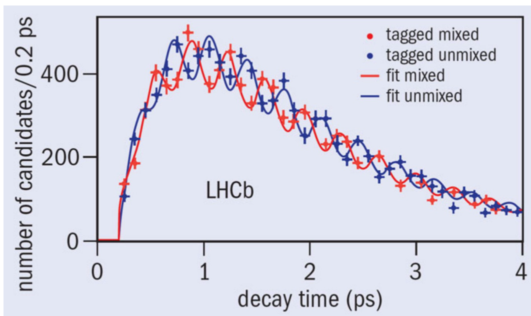

Fig. 1. The decay-time distribution measured for B^0_s oscillations.

This is for B-meson oscillations, $B^0 - \bar{B}^0$. The discussion applies to this system, as well as $D^0 - \bar{D}^0$ mixing.

We have provided a description of meson mixing. Given that both K_L & K_S decay to $\pi\pi$ states, CP is violated in the SM.

we will not discuss how to obtain this prediction in the SM, which depends on the CKM phase.